The increase with energy of the parton transverse momenta in the current fragmentation region and related pQCD phenomena in DIS at very high energies. *

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Abstract

We find, within the pQCD dipole model for DIS processes, a rapid increase with energy of the scale of parton momenta in the current fragmentation region in the limit $s \to \infty$ and fixed Q^2 . We explain the equivalence between the dispersion representation over Q^2 for LT pQCD zero angle amplitude of $\gamma^* + T \to \gamma^* + T$ scattering at large energies and the k_t factorization, and use it to evaluate the scales of the hard processes in the current fragmentation region. We derive within the black disc (BD) regime the modified Gribov formula for the total cross-section of the DIS. We evaluate the coherence length of the processes relevant for the BD regime and find that it increases with energy as $\sim s^{0.6}$ i.e. significantly slower than in the parton model ($1/2m_N x$ - the Ioffe length) as well as in pQCD. In the BD regime we estimate the gluon densities, local in the coordinate space, and find that they do not decrease with the energy. We discuss briefly how the new pQCD phenomena may reveal itself in the proton-proton collisions at LHC.

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I. INTRODUCTION

The aim of this talk is to demonstrate the existence of the new hard QCD phenomena in DIS at small $x = Q^2/s$ and large but fixed Q^2, in the vicinity of the onset of the BD regime, in the kinematic domain where the pQCD is still valid, and to apply them to the physics to be observed at the LHC. We use the dipole model and k_t factorization [1, 2] as a tool and find that in the limit $x_B \to 0$ and large but fixed Q^2 the essential transverse momenta of partons within the dipole are rapidly increasing with energy in contrast with the limit $Q^2 \to \infty$ and fixed x_B. In the case of the longitudinal photon cross-section σ_L the essential region of integration over the transverse momenta of the constituents within the quark dipole is $k_t^2 \approx Q^2/4$ at moderate and large x but increases with the energy at sufficiently small x. Similar effects occur in the case of the transverse photon cross-section σ_T, where the relevant invariant masses increase with the energy even more rapidly. This result is in a sharp contrast with the regime of moderate energies like the energies of HERA ($s \leq 10^5$ GeV^2), where dominant configurations in the cross section $q\bar{q}$ have invariant masses $M^2(q\bar{q}) \leq Q^2$.

The rapid increase of the characteristic transverse scales in the fragmentation region has been discussed in ref. [3], however within the black disk (BD) regime. Our new result is the prediction of the increase of the jet momenta in the fragmentation region, in the kinematical domain where methods of pQCD are still applicable. To visualize analytically the origin of this phenomenon in pQCD we use the double logarithmic approximation which is a rather rough approximation. The calculations show that in σ_L the relevant invariant masses of the $\bar{q} - q$ pairs (dipoles) increase to $M^2 \sim 1.5 - 2Q^2$ for the quark dipole-nucleon interactions at invariant energies $s = 10^5 - 10^6 GeV^2$. Let us stress that this new regime is characterized by the increase of the parton transverse momenta with energy in the photon fragmentation region and thus it is different from the well known phenomena of the diffusion to the large transverse momenta in $\ln k_t^2$ plane in the center of rapidity [4].

We have carried our calculations in the leading logarithmic approximation. However, we expect that qualitatively our results will be valid in the NLO and beyond, because to a large extent the NLO corrections are taken into account in the phenomenological fits that fix the parameters of the parton distributions in the LO approximation. Remember that the parton distributions have been extracted from data in the limit of $Q^2 \to \infty$ and fixed x where effects discussed in the paper should be a correction.

For the deep inelastic lepton scattering off a nucleon we evaluate the dependence on the energy s and transverse momenta Q^2 of the three characteristic scales. The first scale is the maximum in the distribution of jets in the current fragmentation region of the total cross-section over M^2 -M_1^2 . Then we evaluate the region of M^2 that gives the median and the dominant contributions in the cross-section. We first determine the scale M_{t1}^2 that gives 50% of the cross-section, and then determine the characteristic transverse scale that characterizes the "tail", 80% of the total cross-section ,it's upper boundary will be denoted as M_{t2}^2. The value of 80% is chosen to quantify M^2 that is relevant for the process. The more detailed analysis will be given elsewhere.

Another important scale is the scale of the invariant mass of the dipole pair M_b^2 when the partial width for dipole-nucleon scattering reaches one for the central impact parameter $b = 0$. This scale characterizes the onset of the black disc limit, and is close to that obtained in the similar analysis in refs. [18, 19, 20].

As an application of the above discussed results we find that at sufficiently high energies

the variety of coherence lengths substitutes the Ioffe length -$1/2m_N x$ [11, 12] familiar from QED [13, 14]. The derived pattern is different also from the one expected in the application of the DGLAP formalism for the scattering off the photon, where the logs come from the region of integration over the transverse momenta $0 \ll k_t^2 \ll Q^2$. We pay special attention to the coherence length that corresponds to the configurations responsible for the onset of the black disk limit (and dominant in the total cross-section). We show that taking into account the contribution of dipoles with large masses and of black disc limit to explain the probability conservation leads to the coherence length l_c increasing more slowly with the energy. This is the because the coherence length for a given process is

$$l_c = s/(M^2(s) + Q^2), \tag{1.1}$$

where $M^2(s)$ is the typical M important in the inelastic(total) cross section. Since effective M^2 is increasing with the energy -see above discussion - the coherence length for the onset of the black limit is $l_c \leq (s/(M_b^2(s)+Q^2))$, and calculations show that this coherence length increases with the energy much slower than Ioffe length, namely as $l_c \sim s^{0.6}$.

The increase with energy of the M_b^2 scale has been discussed in detail in [3, 18, 19, 21]. However, the slowing of the increase of the coherence lengths with energy in the kinematics corresponding to the onset of the BD regime, the modified Gribov formulae for the structure functions in the BD regime are our new results.

II. THE DIPOLE APPROXIMATION APPROACH.

We use in our calculations the dipole approximation, combined with the k_t factorization. It follows from the QCD factorization theorem [6, 22] that in the LO approximation inelastic cross section of the scattering of the longitudinally polarized virtual photon off a hadron target is calculable in terms of the light-cone wave functions of the virtual photon:

$$\begin{aligned}\sigma &\sim \int \frac{d^2k_t}{2(2\pi)^3}\int d^2r_t dz \frac{1}{2(2\pi)^3}\\ &\times\ \psi(r_t,z)(2\psi(r_t,z)-\psi(r_t+k_t)-\psi(r_t-k_t))\\ &\times\ \frac{4ImT^{ab}_{\mu_1\mu_2}q^{\mu_1}q^{\mu_2}}{s}.\end{aligned} \tag{2.1}$$

Here r_t is the transverse momentum of the constituent within the dipole and k_t is the transverse momentum of the exchanged gluon. The tensor T^{ab} is the sum of the diagrams describing imaginary part of the amplitude for the gluon scattering off the target T. The integral of T^{ab} over d^2k_t is proportional to the gluon structure function of the target T. The function ψ is the wave functions of the virtual photon , and q^μ is the photon momentum.

Within the LO accuracy which we use in the analysis this formula can be rewritten as

$$\sigma \sim \int_0^1 dz \int d^2r_t(\nabla_i\psi(\vec{r}_t,z))^2 x'G(x',4r_t^2). \tag{2.2}$$

Here the derivatives are over r_{it}, $M^2 = r_t^2/(z(1-z))$ is the invariant mass of the dipole, $x' = (M^2+Q^2)/s$. The function $G(x',4r_t^2)$ is the integrated gluon function. The above

equation is the generalization of LO DGLAP, and BFKL approximations to the interaction of the high energety dipole with the wave function given by pQCD, which accounts for the k_t factorization theorem. (Within the DGLAP approximation the structure function xG in the above formulae depends on $Q^2, x = Q^2/s$.)

The above equation has rather general justification at small x, where the LT approximation of pQCD is applicable. Actually it has been understood in preQCD period that in a quantum field theory, where the coherence length significantly exceeds the radius of the target T at large energies, the electroproduction amplitude in the target rest frame is given by a dispersion integral over Q^2 [26]. The pQCD guarantees another general property: in the target rest frame description: the smaller size of the configuration in the wave function of projectile photon leads to a smaller interaction with the target. This property is ensured by the k_t factorization theorem for the interaction of sufficiently energetic dipole with a target. Both properties together give the LO eq.2.2 . In the NLO approximation the structure of formulae is the same except the appearance of the additional $q\bar{q}g, ...$ components in the wave function of photon due to the necessity to account for the QCD evolution of the photon wave function [6].

For the cross section of the interaction of the longitudinal photon, explicitly differentiating the photon wave function, we find:

$$\sigma_L = \frac{\pi\alpha_{\rm e.m.}\sum e_q^2 F^2 Q^2 \alpha_s}{12}\int dM^2 \frac{M^2}{(M^2+Q^2)^4}\cdot x' G(x', M^2). \tag{2.3}$$

Here $F^2 = 4/3$ for the colorless dipoles build of color triplet constituents. The masses of the constituents of the dipole were neglected since we restrict our consideration to the spatially small dipoles only.

Similarly, for the pQCD contribution to the cross-section of the transverse photon we find:

$$\sigma_T = \frac{\pi\alpha_{\rm e.m.}\sum e_q^2 F^2 \alpha_s(r_t^2)}{12}\int dM^2 \frac{(M^4+Q^4)}{(M^2+Q^2)^4}\cdot x' G(x', 4r_t^2). \tag{2.4}$$

Here in the practical use we introduce a cut off $M^2 z(1-z) > u_t^2$, where u_t is a lower cut off, beyond which we can not use the perturbation theory, and in order to obtain the real full cross-section we must add the contribution of the aligned jet model (AJM).

We shall use this equation for the illustrative calculations only, and check it's usefulness by studying the cut-off dependence. The key difference from the longitudinal photon case is that the asymmetry in the z configurations (the aligned jet model) gives significant contribution into the transverse cross-section even at relatively high energies (this contribution dominates in diffractive processes at HERA energies). These contributions correspond to large invariant masses even when the actual transverse momenta are small, since $M^2 = r_t^2/(z(1-z))$, and large for $z \to 0, 1$. We expect however that for sufficiently large energies the relative contribution of the alligned jet model decreases since the contribution of the symmetric pQCD configurations increases with energy much more rapidly. Indeed, the contribution of the low r_t is multiplied by a structure function at the second argument of order $4r_t^2$.

The formulae discussed above in the momentum space obtain the transparent form in the coordinate space description. In the leading order pQCD, the cross-section of the interaction of the longitudinal and transverse photons off the nucleon have the form :

$$\sigma_L(T)(x, Q^2) = \int_0^1 dz \int d^2 d\sigma(d, s, Q^2)|\psi_{\gamma L(T)}(z, d)|^2. \tag{2.5}$$

Here and below d is the transverse diameter of the dipole, and ψ is the light-cone wave function of the photon and σ is the cross section for the quark dipole scattering off target T.

The connection between d^2 and transverse momenta and invariant masses is given by $d^2 = \pi^2/(4r_t^2)$, $M^2 = r_t^2/z(1-z)$ [23]. The cross-section of the dipole -hadron interaction is given by the equation [24, 25] :

$$\sigma(d, x') = F^2(\pi^2/3)d^2\alpha_s(M^2)x'G(x', M^2). \tag{2.6}$$

Here d is the transverse scale of the dipole, and once again $x' = (M^2+Q^2)/s$. The derivation of this formula heavily uses validity of the LO k_t factorization theorem -see the above discussion.

The wave functions of the longitudinal and transverse dipole are given by

$$|\psi_L(z, d)|^2 = (6/\pi^2)\alpha_{e.m.}\sum_1^{n_f} e_q^2 Q^2 z^2(1-z)^2 K_0^2(\epsilon b), \tag{2.7}$$

where $\epsilon^2 = Q^2 z(1-z)$,and

$$|\psi_T(z, d)|^2 = (3/2\pi^2)\alpha_{e.m.}\sum_1^{n_f} e_q^2(z^2 + (1-z)^2)(\epsilon^2 K_1^2(\epsilon d). \tag{2.8}$$

Here K_0, K_1 are the standard zero and first order MacDonald functions.

Note that with the LO accuracy the cross-section representations as integral over transverse momenta and over dipole sizes are just connected by a direct and inverse Fourie transformation.

In the practical application of the dipole model for the longitudinal photon the above formulae can be further simplified since it may be shown that the dominant contribution in the pQCD comes from z=1/2. In this case it is possible to further improve the model, introducing a parameter λ, such that $d^2 = \lambda/M^2$, and determining it from the best fits with the data on HERA structure functions and J/ψ production. For the symmetric configurations, taking into account the above connection between d^2 and r_t^2, we have $d^2 = \pi^2/M^2$. However it is possible to show that the results do not change for the whole region of $\lambda > 4$ [18], and thus are insensitive to the precise value of the coefficient in the formula connecting d^2 and M^2. At high energies a new QCD phenomena appear- the increase with energy of the transverse momenta of constituents within the dipole. This property is absent in the LO, NLO prescriptions for the calculation of Feynman diagrams of pQCD where the contribution of the kinematical region: $M^2 \gg Q^2$ is neglected. In other words, the LO/NLO are internally inconsistent at sufficiently large energies since they ignore the basic property of pQCD-smaller size of dipole-faster increase of a cross section with energy. We explained above that this generalization is necessary to account for basic property of pQCD that the interaction of color neutral dipole of zero transverse size with a target should be 0.

The next practical question is how to parameterize the structure functions for the realistic calculations . Here we shall use CTEQ5L distributions [27]. The CTEQ5L distributions has been shown to be in good agreement with HERA data [19], in the range of $x \sim 10^{-3}, 10^{-4}$. To extrapolate to very small x, we shall use the approximate formulae in the form:

$$xG \sim a(M^2)/x^{c(M^2)}. \tag{2.9}$$

Here the functions

$$a(M^2) = 2.00123 - 1.69772 \cdot 10/M^2 + 3.07651/\sqrt{M^2/10.} - 0.228087 \cdot \log M^2/10., \quad (2.10)$$

$$c(M^2) = 0.045 \log(M^2) + 0.17, \quad (2.11)$$

where M^2 is in GeV^2. This formula is the fit to the observed behavior of the structure functions in HERA for 150GeV$^2 \geq Q^2 \geq 3$ GeV^2 , made by ZEUS and H1 collaborations [28]. We shall use this function also in the kinematics where the CTEQ parton distributions are absent. Note that observed increase of $c(M^2)$ with M^2 can be explained as due to Q^2 evolution of parton distributions cf. ref.[29] and within the resumation approach [9, 10].

III. THE HARD FRAGMENTATION PROCESSES.

We have carried both analytical and numerical analysis of the transverse scales. The analytical calculation has been made for the toy model based on the double logarithmic approximation and will be published in the full version of the paper. The numerical results will be presented below. We shall extrapolate our results to energies of order $s \sim 10^7$ GeV^2. These energies are unattainable due to the fact that there is no e-p collider for such energies. (The proposed e-p DIS facility at the LHC may reach the energies of order 10^5 GeV^2. However these results are interesting both from the theoretical point of view (probing the limits of the pQCD at very high energy), and from the practical point of view, giving the information about the parton distributions at the LHC. The relation of our results to the processes at the LHC will be discussed in section V.

A. Numerical analysis: longitudinal photons.

Let us analyze the integrand in the integral representation of the cross-section 2.5 as the integral over d^2. This integral can be rewritten as integral over $M^2 = 4/r_t^2$, $r_t^2 = (\pi^2/4d^2)$ [23], and we use the dominance of the symmetric configurations $z = 1/2$. This approximation is good since $z(1-z)$ is a slow function of z for a large range of z. With the logarithmic accuracy such integral must numerically be the same as the integral over M^2 in the momentum representation of the corresponding cross-section. We carried our calculations both in the coordinate and momentum representations and found that the results coincide within a 10% accuracy.

Let us first consider the quantity $d\sigma_L/dM^2$. This quantity in the z=1/2 approximation is proportional to the transverse momentum r_t jet distribution in the fragmentation region.

In order to characterize the behavior of the density we calculate two scales: M_1^2 and M_t^2. The first of these scales characterizes the maximum of the density. This scale lies $M_1^2 \leq Q^2$, for all the values of the energy less than 10^{11} GeV^2. In line with the expectations based on the toy model we see that this maximum however slowly increases with energy for every value of Q^2, with the energy dependence for $Q^2 \geq 5$ GeV^2 being approximately $s^{0.04}$.

The scale M_1^2 however does not give a full characterization of the density, the reason is that the rapid decrease of the square of the derivative of the wave function is compensated partly by the rapid increase of the structure function with the increasing M^2. In order to characterize this effect it is natural to determine the new scale M_{t1}^2 that corresponds to the

cut off in M^2 that gives, say, 50% of the total cross-section, and another scale M^2_{t2} that corresponds to 80% of the cross-section and characterizes the magnitude of the tail.

The determination of M^2_{t1} is given in Table 1 for the characteristic virtualities $Q^2 = 5, 10, 20, 40, 100$ GeV^2 and for realistic energies (up to those achievable at LHC). We see that for not very small Q^2 (starting from $Q^2 \sim 10$ GeV^2) and for HERA energies this scale lies beyond $0.75Q^2$, where Q^2 is an external virtuality, thus justifying the use of DGLAP at these energies. We see from the tables that this median scale rapidly increases with energy, and will overcome Q^2 at $Q^2 = 10 - 20$ GeV^2 already at LHC energies and may significantly overcome Q^2 if we shall go beyond LHC.

Let us consider the second scale, M^2_{t2}. We see that the tail exists even at HERA energies for $Q^2 < 200$ GeV^2, when the M^2_{t2} exceeds Q^2. The tail also increases with the increase of energy, even more rapidly then the median scale M^2_{t1}.

The scale M^2_{t2} is always larger than Q^2. If we extrapolate to the very large (though unrealistic) energies we see that it exceeds Q^2 for very small $Q^2 \sim 5GeV^2$ by a factor of 5. In other words, for sufficiently small momenta and high energies the leading logarithmic approximation leads to the very long tails, that are strictly beyond the control of usual prescriptions for DGLAP and BFKL approximations and must be taken into account using k_t factorization theorem, at least in the LO approximation.

We conclude that for sufficiently small external virtualities and high energies (large 1/x) one cannot use the naive DGLAP approximation where transverse momenta in the upper rung of the ladder are $\ll Q$, while the the dipole approximation provides a reasonable description of the cross-sections. In this approximation the LO DGLAP/resummed ladder gives the cross-section of the dipole scattering of the target.

B. The evaluation of the transverse scale: transverse photons.

The same two scales M^2_t and M^2_1 that we defined for longitudinal photon can be defined for the transverse photon.

In this case however the use of the pQCD expression poses a problem due to a potential large contribution of the nonperturbative physics, described by the aligned jet model. Another difference from the longitudinal photon case is the large contribution of the asymmetric configurations, with $z \neq 1/2$. These contributions are dominant in the AJM model, and lead to large invariant masses even for small transverse momenta, due to relation $M^2 = r_t^2/(z(1-z)$. In this talk we are concerned only with the contribution of the pQCD. In order to exclude the contribution of the AJM (nonperturbative QCD) we impose a cutoff $M^2 z(1-z) > \Lambda^2$, which removes the contribution of the low transverse momenta. The use of the pQCD formulae, even for illustrative purposes, would be justified only if the cross section depends weakly on the cutoff. We see however, that for $Q^2 < 20$ GeV^2 even for the energies of the order 10^7 GeV^2 there is a strong dependence (of the order 20%) of the maximum of the curve $d\sigma/dM^2$ on a very small change of the cutoff (from $\Lambda = 0.75$ to $\Lambda = 1$ GeV^2. The data for the median and the tail transverse scales M^2_{t1}, M^2_{t2} are presented in Tables 3, 4. We see that for HERA energies there is a dependence of order 10% of these scales on the cut off even at $Q^2 \sim 100$ GeV^2. This dependence however decreases with energy.

The detailed analysis of the relative contribution of the pQCD and aligned jet model will be given in ref. [31]. In tables 4, 5 we presented our results for transverse scales at $Q^2 = 20, 40, 100$ GeV^2. We considered the integrand for cross-section as a function of the invariant mass and cut off, after integrating over permitted z for given invariant mass and

cut off.

The scale $M_1^2 < Q^2$ for all energies and $Q^2 > 10$ GeV^2 behaves similar to the longitudinal photon, However this scale is shifted to smaller invariant masses relative to longitudinal photons. For every given virtuality the scale continues to increase with the increase of energy, with approximately the same rate as for longitudinal photons.

The characteristic feature of the invariant mass distribution for the transverse photons is however that it becomes much broader, and increases with energy more rapidly than for longitudinal photons. As a result although the transverse scales for HERA energies are slightly smaller than for longitudinal photons, for high energies, achievable at LHC they already overcome them. The rate of increase is $M^2 \sim s^{0.1}$ for LHC energies and $Q^2 \sim 50$ GeV^2.

It is once again instructive to look where $Q^2 = M_t^2$ as a function of energy. We see that the tail starts (with logarithmic accuracy) for higher Q^2 than for the case of the longitudinal photons.

Since our results show qualitative stability with the change of the cut off and the dependence of cut off (and AJM contribution) decrease with Q^2 and energy, we expect that our results remain qualitatively the same if we shall take into account the AJM contribution explicitly.

Let us note that the appearance of the tails at sufficiently large energies (small x) can be expected from the comparison with the Gribov formulae for the BD regime[26]. Indeed, assuming a smooth matching of the pQCD and BD regime the perturbative distribution must match the black limit spectral density , and this density leads to the tail with masses increasing with energy. Thus the existence of a large mass tail beyond the naive DGLAP approximation seems to be a necessary property of pQCD near the black limit.

Finally, we expect that our results will not change in the NLO/resumed model. The reason is that NLO effects are partly taken into account in the gluon distribution fits to experimental data.

C. The evaluation of the black scale.

We can now combine the constraint for the partial wave at zero impact parameter to become unit with the CTEQ5 structure function to determine the invariant masses that correspond to the onset of the black limit.

The typical dependence of the black limit onset scale M_b^2 on energy is presented in Table 3. for the gluonic dipole. We consider two cases: one when the partial wave Γ at the central impact parameter reaches 1, another when it reaches 1/2 [3]. Indeed, when the partial wave reaches 1/2 the probability of inelastic interactions reaches 3/4, i.e. interactions become strong and pQCD can not be used any more [3].

The characteristic transverse momenta that corresponds to the black scale for the gluonic dipole for the energies $s = 10^6 - 10^7$ GeV^2 are $2 - 2.5$ GeV/c if we impose $\Gamma = 1$ condition or $4 - 4.5$ GeV/c if we impose $\Gamma = 1/2$. Note that for HERA energies ($s = 10^4$ GeV^2 we do not obtain the black limit for gluonic dipole for $Q^2 > 40$ GeV^2, while for lower values of Q^2 we obtain formally $k_t \sim 1$ GeV, the value that seems beyond the limits of the applicability of the method we used here to determine this scale.

For fermionic dipoles we see that there is no black limit for HERA energies, although nonperturbative effects (corresponding to $\Gamma = 1/2$) seem to start to appear for $k_t < 1$ GeV. For the energies $s \sim 10^6 - 10^7$ GeV^2 the transverse scale is $\sim 2 - 2.5$ GeV for the partial

wave $\Gamma = 1$ condition, $3 - 3.5$ GeV respectively for $\Gamma = 1/2$ conditions. These results are in good agreement with the previous determination of these scales [3, 18, 20].

For our purposes it will be important to determine the rate of increase of the M_b^2. This rate does not depend on the type of the dipole (fermionic or gluonic) or on the partial wave condition.

For the gluonic dipole in DIS for s up to $s \sim 10^7$ GeV^2 the black limit scales increase relatively slow as $s^{0.3}$. However at the energies above $s \sim 10^7$ GeV^2 the increase speeds up with $M^2 \sim s^{0.4}$ starting from $s \sim 10^8$ GeV^2 energies (for the exponent for partial wave 1/2 we obtain 0.39, for the exponent with partial wave equal to 1 we obtain 0.38), and small $Q^2 < 40$ GeV^2. Note that this region is where we expect that there is no dip influence.

We show that for moderately large Q^2, where the coupling constant is small the cross-section will be dominated by the black limit.

We found from this subsection that for the DIS processes at very high energies there exist 3 regimes. First the nonperturbative black disk regime. This regime is valid for invariant masses up to M_b^2, then the standard pQCD regime described by DGLAP and then the new pQCD regime, where one can not use DGLAP, BFKL resummed models directly, but must combine them with the dipole approximation. In this new regime a virtual photon creates a parton pair with invariant mass larger than the external virtuality, that scatters off the target by the DGLAP/resummed ladder. Thus in the s$-Q^2$ plane we find three regions: the black limit region-the virtualities where the cross-section is dominated by the black limit, the region where the cross-section is dominated by the usual pQCD, and the new regime area where the tail gives a significant contribution to cross-section.

D. A hadron structure function in the black disc limit

The above formulae make possible first quantitative evaluation of the numerical coefficient in the asymptotical expression for the structure function of DIS in the black disc limit suggested in ref. [5, 7].

The asymptotic behavior of the hadron structure functions at $s \to \infty$ in QCD is rather close to Gribov formulae for the total cross section of DIS derived long ago for heavy nuclear target in the black disc limit [26]. In the case of the transverse photon

$$F_2 = 2\pi R_T^2(s) Q^2 \kappa \rho(M^2 \to \infty)(\ln(x_0/x)), \tag{3.1}$$

where ρ is the spectral density for the transition $e^+e^- \to$ hadrons :

$$\rho(M^2) = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}. \tag{3.2}$$

The density $\rho(M^2)$ tends to constant at large invariant masses. R_T is the maximal impact parameter, where the partial wave reaches 1. Fourie transform of two gluon form factor of a nucleon: $F = 1/(1 + q^2/\mu^2)^2$ measured in the hard diffractive processes at HERA and at FNAL describes dependence of a partial amplitude f on the impact parameter b : $f(b, x) \propto (1/x)^\lambda exp^{-\mu b}$. Here $\mu^2 \approx 1GeV^2$. In the LT approximation the structure function of a hadron in DIS is usually parametrized as $F_2 \sim (1/x)^\lambda$. Finally we obtain:

$$R_T = \lambda(M_b^2) ln(x_0/x)/\mu \tag{3.3}$$

Here $x_0 \sim 10^{-2}$, A slightly more complicated reasoning which uses completeness over the hadronic states shows that Eq.3.1 is valid even if to account for the gluons and quark-antiquarks pairs in the wave function of the dipole. The coefficient κ in eq. 3.1 is determined by the upper limit of the integration over M^2 and is equal to $M_b^2 \sim s^\kappa$. Our results show that this coefficient weakly depends on energy, with $\kappa = 0.3 - 0.4$ at the LHC energies and beyond.

IV. THE COHERENCE LENGTH

In the previous sections we determined the absolute values and the energy dependencies of the effective transverse scale and the black limit scale at high energies which allowed us to evaluate coherence length.The coherence length l_c corresponds to the time, such that the dipole fluctuation exists this time interval at a given energy. The original suggestion of the existence of the coherence length in the deep inelastic scattering was first made by Ioffe, Gribov and Pomeranchuk [11, 26] It was found already in the sixties by Ioffe [12] that the coherence length at moderate x_B is $l \sim 1/2m_N x_B$ i.e. it linearly increases with energies. At higher energies we have a whole variety of local coherence lengths. Here we consider the coherence length, that corresponds to configurations near the onset of the black limit that dominate the cross-section at very high energies.

Since near the onset of the black limit $M^2 \sim s^{0.4}$ we conclude, that in the vicinity of black limit i.e. for sufficiently high energies this coherent length increases like $l_c \sim s^{0.6}$, i.e. it increases with energy much slower than for lower energies, when Ioffe result is applicable.

The important characteristic of the hard processes is the local gluon density produced in the hard high energy process. Rough estimate of the three dimensional space density of gluons (number of gluons in a unit of the space volume) gives

$$n \sim N/(l_c\pi/r_t^2) > xG(x, 4r_t^2)/(l_c\pi/r_t^2), \tag{4.1}$$

where N is a total number of the emitted gluons.

At the LHC energies the coherence length increases with energy as $\sim s^{0.6}$ for sufficiently small Q^2 ($\sim s^{0.55}$ near the energies of order 10^{11} GeV^2). Using known dependence of the gluon structure function on energy, we obtain that the number of gluons increases like $s^{0.4}$, while the same formulae extrapolated to superhigh energies gives $s^{0.5}$. This means, that the naive estimate gives slowly decreasing gluon number density $n \sim s^{-0.2} - s^{-0.1}$. Here, however, we neglected the increase of the transverse momenta that may also contribute to the increase of a local gluon density. The realistic estimate of this increase is however impossible in the LO approximation. Consequently, within our accuracy we cannot determine whether the local (three-dimensional) gluon density is increasing or slowly decreasing. To address this question more quantitatively we have to go beyond the naive estimate and perform a detailed calculation of the local density, taking into the account the inhomogeneity of the ladder. Such analysis will be done elsewhere.

We conclude that there are indications that the local gluon density may appear large in the black disc limit regime.

The opposite example, of high light cone densities and low local density rapidly decreasing with energy was considered some time ago by Mueller in QED [30]. However in that case there is no rapid increase of interaction with energy.

It is worth noting that discussed above pattern for the energy dependence of the coherence length leads to a change of the structure of the fast hadron wave function as compared to the

Gribov picture where the longitudinal size of the hadron is determined by the wee parton cloud and energy independent $\sim 1/\mu$ where μ is the soft scale. On the other hand a slower rate of the increase of the coherent length with energy than $1/m_N x$ leads to decrease of the longitudinal size of the hadron with energy. The typical size is determined by the BD momentum at a given impact parameter for a particular energy. Moreover since the BD momentum is larger for small impact parameters the nucleon has a form of a concave lens. It is of interest also that for the zero impact parameter the longitudinal size of a heavy nucleus is smaller than for a nucleon.

V. EXPERIMENTAL CONSEQUENCES

The current calculations of cross-sections of hard processes at the LHC are based on the use of the DGLAP parton distributions and the application of the factorization theorem. Our results imply that the further analysis is needed to define the kinematic regions where one can use DGLAP distributions. We showed in the paper that for DIS at high energies there are kinematic regions where one is forced to use a k_t factorization and the dipole model instead of direct use of DGLAP. A similar analysis must be made for the pp collisions at LHC. This analysis is however more complicated since the proton can not be approximated by a dipole and thus the DIS results for the same energies and external virtualities can not be transferred directly to *pp* case. This is a problem for a future work.

The hard processes initiated by the real photon can be directly observed in the ultraperipheral collisions [20]. The processes where a real photon scatters on a target, and creates two jets with an invariant mass M^2, can be analyzed in the dipole model by formally putting $Q^2 = 0$, while M^2 is an invariant mass of the jets. Then with the good accuracy the spectral density discussed above will give the spectrum of jets in the fragmentation region. Our results show that the the jet distribution over the transverse momenta will be broad with the maximum moving towards larger transverse momenta with increase of the energy and centrality of the γA collision.

Finally, our results can be checked directly, if and when the LHeC facility will be built at CERN.

[1] S. Catani, M. Ciafaloni, F. Hauptmann, Nucl. Phys. B366 (1991) 135.
[2] J. Collins, K. Ellis, Nucl. Phys., B360 (1991) 3.
[3] L. Frankfurt, M, Strikman and C. Weiss, Ann.Rev.Nucl.Part.Sci.55:403-465,2005.
[4] E. Kuraev, V. Fadin, L. Lipatov, Sov. Phys.-JEP, 44 (1976) 443; 45 (1977) 199. I. Balitsky and L. Lipatov, Sov. J. Nucl. Phys., 28 (1978) 822.
[5] H. Abramowicz, L. Frankfurt and M. Strikman, Surveys in High Energy Physics, 11 (1997) 51.
[6] L. Frankfurt and M. Strikman, Phys. Rept., 160 (1988) 235.
[7] V. Guzey, L. Frankfurt, M. Strikman,M. McDermott, Phys.Rev.Lett., 87 (2001) 192301.
[8] G. Altarelli and G.Parisi, Nucl. Phys., B126 (1977) 298; V.N. Gribov and L. N. Lipatov, Sov. J. of Nucl. Phys., 15 (1972) 438,672; Yu.L. Dokshitser, Sov. Phys. JETP 46 (1977) 641.
[9] G. Altarelli, S. Forte, R.D. Ball, Nucl. Phys., B621 (2002) 359; B674 (2003) 459.
[10] M. Ciafaloni, P. Colferai, G.P. Salam, A. M. Stasto, Phys. Lett., 587 (2004) 87; Phys. Rev. D68 (2003) 114003.
[11] L. B. Ioffe, V. Gribov, I. Pomeranchuk, Sov. J. of Nucl. Phys., 2 (1966) 549.
[12] B. L. Ioffe, Phys. Lett., B30 (1969) 123.
[13] L. Landau and I. Pomeranchuk, Dokl. Akad. Nauk SSSR 92 (1953) 535,735
[14] M.L. Ter-Mikaelyan, Zh. Exp. Theor. Phys., 25 (1953) 296.
[15] L. McLerran and R. Venugopalan, Physical Review, D49 (1994), 2233,3352; D50 (1994) 2225; D59 (1999) 094002; see also L. McLerran, Surveys High Energy Phys., 18 (2003) 101; Nucl. Phys. A702 (2002) 49, for the latest reviews on the subject.
[16] A. H. Mueller Nucl. Phys. B **415**, 373 (1994).
[17] Parton saturation: An overview A. H. Mueller arXiv:hep-ph/0111244 Lectures given at Cargese Summer School on QCD Perspectives on Hot and Dense Matter, Cargese, France, 6-18 Aug 2001.
[18] T. C. Rogers, A. M. Stasto and M. I. Strikman, Unitarity Constraints on Semi-hard Jet Production in Impact Parameter Space,"arXiv:0801.0303 [hep-ph].
[19] T. C. Rogers and M. I. Strikman, Hadronic interactions of ultra-high energy photons with protons and light nuclei in the dipole picture," J. Phys. G **32**, 2041 (2006)
[20] L. Frankfurt and M. Strikman, in Phys. Reports, 455 (2008) 105.
[21] V. Guzey, L. Frankfurt, M. Strikman, M.McDermott, Eur.J. of Physics, C16 (2000) 641.
[22] L. Frankfurt, A.Radyushkin, M. Strikman, Phys. Rev. D55(1997) 98.
[23] L. Frankfurt, W. Koepf, M. Strikman, Phys. Rev. D57(1998) 512.
[24] L. Frankfurt, G.A. Miller, M. Strikman, Ann. Rev. Nucl. Part. Sci., 44 (1994) 501.
[25] B. Blaettel, G. Baym, L. Frankfurt and M. Strikman, Phys. Rev. Lett., 70 (1993) 896.
[26] V. N. Gribov, Sov. Phys. JETP 30 (1970) 709.
[27] CTEQ collaboration,hep-ph/0006300.
[28] C. Adloff et al, Phys. Lett., B520 (2001) 183; Chekanov et al, Nucl. Phys. B713 (2005) 3.
[29] Yu. Dokshitzer,D. Diakonov and S. Troyan, Phys. Reports, 58 (1980) 269.
[30] A.H. Mueller,Nucl.Phys.B307 (1988) 34.
[31] B. Blok, L. Frankfurt and M. Strikman, in preparation.

Table 1. The scale M_{t1}^2 (50% of the total cross-section) for longitudinal photons in DIS

	$Q^2 = 5$ GeV^2	$= 10$ GeV^2	$= 20$ GeV^2	$= 40$ GeV^2	$= 100$ GeV^2
$s = 10^4$ GeV^2	6.5 GeV^2	9 GeV^2	16 GeV^2	28 GeV^2	60 GeV^2
$s = 10^5$ GeV^2	7 GeV^2	10.5 GeV^2	17.5 GeV^2	31 GeV^2	68 GeV^2
$s = 10^6$ GeV^2	7.5 GeV^2	11 GeV^2	19 GeV^2	34GeV^2	77GeV^2
$s = 10^7$ GeV^2	8 GeV^2	12 GeV^2	21 GeV^2	37 GeV^2	87 GeV^2

Table 2. The scale M_{t2}^2 (80% of the cross-section)for longitudinal photons in DIS

	$Q^2 = 5$ GeV^2	$= 10$ GeV^2	$= 20$ GeV^2	$= 40$ GeV^2	$= 100$ GeV^2
$s = 10^4$ GeV^2	10.5 GeV^2	17 GeV^2	30 GeV^2	54 GeV^2	120 GeV^2
$s = 10^5$ GeV^2	11.5 GeV^2	19 GeV^2	34 GeV^2	60 GeV^2	140 GeV^2
$s = 10^6$ GeV^2	12.5 GeV^2	21 GeV^2	38 GeV^2	67GeV^2	160GeV^2
$s = 10^7$ GeV^2	14 GeV^2	23 GeV^2	42 GeV^2	75 GeV^2	180 GeV^2

Table 3. The scale M_{t1}^2 for transverse photons in DIS.

	$Q^2 = 20$ GeV^2	$= 40$ GeV^2	$= 100$ GeV^2
$s = 10^4$ GeV^2	24(20) GeV^2	31(26) GeV^2	50(45) GeV^2
$s = 10^5$ GeV^2	27(23) GeV^2	37(33) GeV^2	59 (55)GeV^2
$s = 10^6$ GeV^2	32(28) GeV^2	43(38)GeV^2	73 (70)GeV^2
$s = 10^7$ GeV^2	38(33) GeV^2	52 (48) GeV^2	90 (86) GeV^2

Table 4. The scale M_{t2}^2 for transverse photons in DIS

	$Q^2 = 20$ GeV^2	$= 40$ GeV^2	$= 100$ GeV^2
$s = 10^4$ GeV^2	73 (65) GeV^2	100 (90) GeV^2	160 (150) GeV^2
$s = 10^5$ GeV^2	83 (75) GeV^2	120 (110) GeV^2	200 (190) GeV^2
$s = 10^6$ GeV^2	96 (88) GeV^2	140 (130) GeV^2	260 (250) GeV^2
$s = 10^7$ GeV^2	110 (100) GeV^2	180 (170) GeV^2	330 (320) GeV^2

Table 5. The scale k_t for the onset of the black disk regime.

	$\Gamma = 1$	$\Gamma = 1/2$
$s = 10^4$ GeV^2	1 GeV	1.6 GeV
$s = 10^5$ GeV^2	1.6 GeV	2.3 GeV
$s = 10^6$ GeV^2	2.3 GeV	3.2 GeV
$s = 10^7$ GeV^2	3.3 GeV	4.5 GeV

The values of M^2 here in the tables 3,4 correspond to r_t cut offs 1 GeV, and 0.75 GeV (in the brackets)